A GENERAL ANALYSIS ON THE TIMING JITTER IN D/A CONVERTERS

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ABSTRACT

A general analysis on stochastic timing errors (clock or timing jitter) is presented for Digital to Analog Converters (DAC). The obtained results describe the effects of (non)correlated errors for given signal properties, and reveal the nature of the tradeoff between oversampling ratio, resolution and noise shaping in the context of noise-shaped DACs and Continuous-Time (CT) Sigma Delta ($\Sigma\Delta$) ADCs. The importance of timing jitter for wideband DAC performance is exemplified with theory and simulations.

1. INTRODUCTION

One of the fundamental problems of DACs is the timing accuracy of the conversion. The discrete-time signal $w(m)$ is converted to a continuous-time signal $s(t)$ at a clock rate $f_s = 1/T_s$. An N bit DAC consists of $2^N - 1$ equally designed elements, eg. current sources, that are switched on/off dependent on the sample that is converted. The switched elements should turn on/off at the same time moment as defined by the ideal clock signal but in reality *timing errors* occur that concern the clock period or the local clock signals that drive the clocked elements.

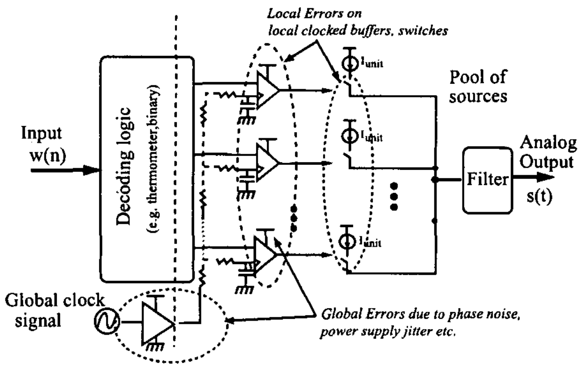

Fig. 1. Physical model of a DAC core (current based).

According to [1]-[3], the timing problem is separated in two major classes; the *global* and the *local*. Timing jitter belongs to the global class of errors and it can be stochastic or deterministic in respect to time. For example, the phase noise of the clock generator [4] has a stochastic nature while power supply jitter [5] is semi-random or deterministic. The local case [1]-[3] is related with the lack of timing in the individual clocked units caused by physical mismatches of the switches, different RC time constants of interconnections etc. In the global case, the timing errors are the same for all the local switching elements at each sample moment mT_s while in the local case they are different for each element. Fig. 1 provides insight to these problems.

For sampled data systems, global-stochastic timing errors have been analyzed in literature in [6]-[14] and elsewhere. The problem has been addressed in several hierarchical levels, such as signal [6, 7], architecture [9]-[13], circuit [14], etc. Studies of jitter in Flash ADCs [9] and Discrete-Time (DT) $\Sigma\Delta$ ADCs [10, 12, 13] show that jitter occurs at the input sample-and-hold circuit of an ADC. From a theoretical standpoint, this type of jitter, named *sampling* or *aperture* jitter, has been analyzed in [6] for stochastic and deterministic signals. On the other hand, jitter in DACs is associated with architectures like Nyquist DACs [8] and CT $\Sigma\Delta$ ADCs [10]-[12]. For 1-bit CT $\Sigma\Delta$ ADCs [10]-[12], it has been shown that the DAC core of the feedback loop is mainly responsible for jitter errors. In addition, developments of Multi-bit DT $\Sigma\Delta$ ADCs [13] bring up the interest to investigate the feasibility of Multi-bit CT $\Sigma\Delta$ ADCs. This suggests a more thorough investigation of the timing errors of DACs and relation with architectural choices. However, no real distinction is made between ADC and DAC on the way the jitter problem is addressed. Usually, the time average of a Taylor expanded, jitter-perturbed, continuous-time sinusoid is used. This results to a misconception of the fundamental difference between ADC-DAC jitter; the DAC's input signal discrete-time derivative is assumed equal to the wanted analog signal's continuous-time derivative. The discrete-time derivative contains all the architectural information, because the level of resemblance to the continuous-time derivative of the wanted signal is determined by the resolution, oversampling ratio and noise-shaping order. Consequently, the tradeoff between jitter properties, resolution, oversampling and noise shaping is not revealed.

The aim of this contribution is to analyze global stochastic timing errors and to reveal the role of the architectural parameters in the performance. A general solution is found that allows the modeling of several jitter types such as Gaussian jitter etc. Resolution, oversampling, noise-shaping and filtering are taken into account and are discussed. Finally, an attempt is made to obtain analytical insight on the effects of supply/substrate jitter [5] in DACs. The results are verified with simulations based on matlab code that emulate the DAC operation subjected to timing errors.

2. THE GENERAL SOLUTION

In the global case the samples are converted at the moments $mT_s + \mu_m$ where μ_m are random correlated or uncorrelated. The input $w(m)$ represents an $N-$bit ideally quantized and sampled signal

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$x(t)$, hence $w(m) = xq(mT_s)$ or, in addition, an L-th order noise shaped version of $xq(mT_s)$. During conversion, the values $w(m)$ are not changed due to μ_m as it happens in the case of A/D sampling, instead, μ_m modulate the signal in a pulse duration fashion [10]. Consequently, the output $s(t)$ is

$$s(t) = u(t) \otimes {}^1\sum_n \Delta w(m)\delta(t - mT_s - \mu_m) = u(t) \otimes y(t) \quad (1)$$

where $\Delta w(m) = w(m) - w(m-1)$ is the derivative of the signal and $u(t)$ is the step function. The model is shown in fig. 2.

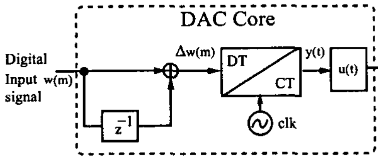

Fig. 2. Ideal DAC core model.

A top-down approach is used in the analysis. First the *general* output power spectrum is found based on general assumptions for correlation, signal etc. Next, examples are examined leading to degenerated forms of this solution.

First we calculate the mean of the empirical autocorrelation[2]. The probabilistic autocorrelation is $R_y(t,\tau) = E\{y(t)y(t+\tau)\}$, where $E\{\}$ defines the expectation in respect to a probability density function (*pdf*) $c(t)$ and the empirical autocorrelation is given by the time average $\hat{R}_y(\tau) = \langle y(t)y(t+\tau)\rangle$. Applying a Fourier transform[3] to $\langle R_y(t,\tau)\rangle$ with respect to τ, we obtain the power spectral density (PSD) of $y(t)$ noted as $S_y(f)$. Here, the process $\{\mu_m\}$ is strictly stationary although for most cases wide sense stationary is enough. Finally, $w(m)$ represents a real waveform.

Assuming a general form of correlation $r_\mu(m,q)$ for μ_m, the time average of the probabilistic autocorrelation is

$$E\{\hat{R}_y(\tau)\} = \frac{1}{T_s}\hat{R}_{\Delta w}(0)\delta(\tau) + \frac{1}{T_s}\sum_{q\neq 0}\hat{R}_{\Delta w}(0)k_q(\tau) \quad (2)$$

where $k_q(\tau) = \int_{-\infty}^{\infty} c_q(t+\tau-qT_s,t)dt$ and $c_{n-m}(t_n,t_m)$ is the joint probability function of the jitter. In order to evaluate the Fourier transform $K_q(f)$ of $k_q(\tau)$ we take use of the double Fourier integral of the jitter $C_{k-l}(f_k,f_l)$ and we find $K_q(f) = e^{-j2\pi fT_s}C_q(f,-f)$ for $q \neq 0$. Consequently, the PSD becomes

$$S_s(f) = \frac{|U(f)|^2}{T_s}\sum_q \hat{R}_{\Delta w}(q)C_q(f,-f)e^{-j2\pi qfT_s} \quad (3)$$

Next, the empirical autocorrelation of the signal's derivative is

$$\hat{R}_{\Delta w}(q) = \langle\Delta w(m+q)\Delta w(m)\rangle =$$
$$\hat{R}_w(m,q) - \hat{R}_w(m-1,q) - \hat{R}_w(m-1,q+1) - \hat{R}_w(m,q-1) \quad (4)$$

When the signal is given by $w(m) = \Re\{\sum_{k=1}^{\infty} A_k e^{j\omega_k mT_s}\}$, we find easily that $\hat{R}_w(m,q) = \Re\{\sum_k |\alpha_k|^2 e^{j\omega_k qT_s}\}$ and,

$$\hat{R}_{\Delta w}(q) = \sum_{k=1}^{\infty} 2A_k^2 \sin^2(\frac{\omega_k T_s}{2})\cos(\omega_k qT_s) \quad (5)$$

Obviously, $\hat{R}_{\Delta w}(0)$ is the power of $\Delta w(m)$.

[1] $\sum_n = \sum_{n=-\infty}^{\infty}$ from hereafter

[2] We use the notation of [15] regarding the forms of autocorrelation.

[3] If the random process $y(t)$ is regular, then $E\{\hat{R}_y(\tau)\} = \langle\hat{R}_y(t,\tau)\rangle$

2.1. Pure "white" timing errors

A case where the timing errors are assumed white is when we address the phase noise effects of clock generators[4] in sampled systems. In this case, $c_{n-m}(t_n,t_m) = c_n(n)c_m(m)$ applies and eq. (3) is transformed to

$$S_s(f) = \frac{|U(f)|^2}{T_s}\hat{R}_{\Delta w}(0)(1 - |C(f)|^2) +$$
$$\frac{|U(f)|^2|C(f)|^2}{T_s}\sum_q \hat{R}_{\Delta w}(q)e^{-j2\pi qfT_s} = S_J(f) + S_W(f) \quad (6)$$

The term $S_W(f)$ contains the signal and the quantization part. Both are mirrored around the sampling carrier and they are attenuated by the jitter and the pulse-hold function through $|C(f)|^2$ and $|U(f)|^2$, respectively. Part of the discrete-time signal's power is shaped to a continuous noise PSD but due to the lack of correlation in the jitter no additional components appear. The term $S_J(f)$ is the noisy part, which can be practically constant for the frequency range of interest. $S_J(f)$ is analogous to $\frac{T_s\hat{R}_{\Delta w}(0)}{T_s^2}$, the PSD of the input signal's derivative, and depends on the jitter through $1-|C(f)|^2$. It is important to note that $\frac{\hat{R}_{\Delta w}(0)}{T_s^2}$ is the discrete-time equivalent of the average derivative power of the continuous-time signal $\langle(\frac{dx(t)}{dt})^2\rangle$. In the *optimal* case $\frac{\hat{R}_{\Delta w}(0)}{T_s^2} = \langle(\frac{dx(t)}{dt})^2\rangle$ applies. Hence, for certain jitter statistics, the best case jitter power is determined by the average power of the continuous-time signal derivative. However, the combination of finite oversampling, finite resolution and noise shaping makes the discrete-time derivative to deviate from the continuous one. This is the basic distinction of the DAC and ADC types of jitter.

For periodic $w(n)$, eq. (5) is used and the power spectrum is

$$S_s(f) = \frac{1-|C(f)|^2}{T_s(\pi f)^2}\sum_{k=1}^{\infty}\frac{A_k^2}{2}\sin^2(\pi f_k T_s) +$$
$$\frac{|C(f)|^2}{(\pi f_k T_s)^2}\sum_q\sum_{k=1}^{\infty}\frac{A_k^2\sin^2(\pi f_k T_s)}{4}\delta(f \pm f_k - qf_s) \quad (7)$$

Consider White Gaussian jitter with $(0,\sigma)$, hence $|C(f)|^2 = e^{-4\pi^2 f^2\sigma^2} \approx 1 - 4\pi^2 f^2\sigma^2$ and an input signal with a single tone. Then $k = 1$ is the fundamental and $k \neq 1$ with $f_k = kf_1$ and k odd stand for the quantization harmonics [17]. Define also $f_s = 2\mathrm{BW}\cdot\mathrm{OSR}$, where OSR is the oversampling ratio. With these assumptions, one can get the SNR in respect to the input signal:

$$\mathrm{SNR} = \frac{A^2}{16\sigma^2\mathrm{BW}^2\mathrm{OSR}\sum_{k=1,\mathrm{odd}}^{\infty}A_k^2\sin^2(\frac{\pi k f_1}{2\mathrm{BW\,OSR}})} \quad (8)$$

Eq. (8) gives the jitter noise as a function of the variance of the jitter, the sample rate f_s (or the signal bandwidth and the oversampling ratio), the spectral shaping of the unit pulse, the signal and quantization components amplitude and frequency. There are several scenarios that one can assume and with the help of eq. (6) to arrive to the desired estimation of the jitter noise. With the assumptions OSR= 1, $f_1 << 1$ and neglecting the quantization power, the Taylor approximated results of [8] and others match eq. (8), leading to an SNR dependent only on the input signal frequency f_1 and the jitter variance σ^2. This is a typical case where the DAC jitter equals the A/D sampling jitter. As

[4] More accurate modeling would assume colored white properties.

$f_1 \to$ BW the SNR saturates to $\frac{1}{4\sigma^2 f_s^2}$, i.e. $\pi^2/4$ or 3.9 dB less than what a Taylor approximation predicts. This occurs because of the S/H nature of the output signal. A typical comparison of theory and simulations is presented in fig. 3(a). A Nyquist-rate DAC (OSR=1) with high resolution ($N = 12$) is considered, so as the quantization part can be safely neglected. A jitter spread of $\sigma = 0.001T_s$, i.e. 2psec in 0.5 GHz clock, allows an SNR of 84 dB at $f_1 = 0.01 f_s$ = 5MHz but when the input frequency becomes equal to $f_1 = 0.1 f_s$ = 50MHz, the SNR drops to 64 dB!

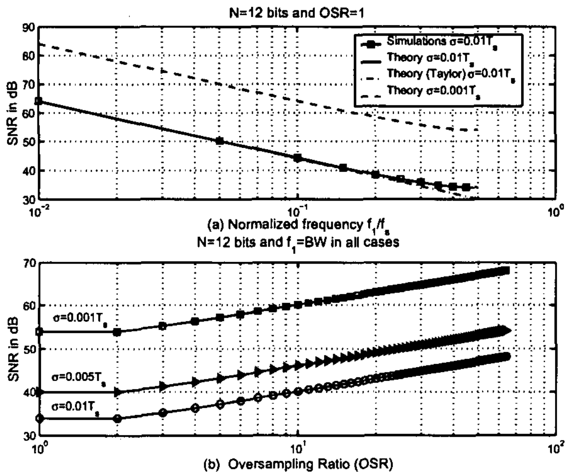

Fig. 3. (a) SNR vs. signal frequency for several σ and (b) SNR vs. OSR with $N = 12$.

Oversampling ratio effects are shown in fig. 3 using eq. (8). The SNR increases 3-dB when OSR doubles since the squared sinusoidal term in (8) dominates the single OSR. For large OSR,

$$\mathrm{SNR} = \frac{\mathrm{OSR}}{4\sigma^2\pi^2 f_1^2} \tag{9}$$

The neglection of quantization effects (very large resolution) combined with absence of noise shaping gives a best case performance, which is met only in Nyquist-rate DACs.

However, systems such as noise-shaped DACs and CT $\Sigma\Delta$ ADCs use a small resolution internal DAC and, consequently, the role of resolution and noise shaping becomes important. In eq. (8) the relative power of the quantization-jitter to the signal-jitter terms was shown. This equation does not show clearly the trade-offs for low N with the presence of noise-shaping. For example, eq. (8) says that without noise-shaping, even for low N around $3-5$, the jitter-signal term is dominant over the jitter quantization term. Then, results and observations saying that when N increases from 1 to several bits the total jitter power is reduced significantly are difficult to explain (see [16]).

More constructive insight to this issue is obtained from eq. (6) recalling $\hat{R}_{\Delta w}(0)/T_s$. We sample and quantize a full scale sinusoid for several OSR and N levels. From the resulting bit-stream, without noise shaping, we estimate with simulations the power $\hat{R}_{\Delta w}(0)/T_s$ and the results are plotted in fig. 4 for two values of the input frequency. We see that for each value of f_1 increasing the OSR gives the well known 3dB per $\times 2$ OSR *only* if

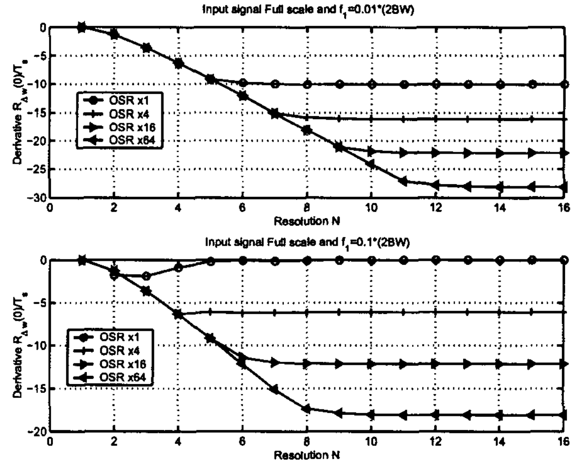

Fig. 4. PSD of discrete derivative normalized to its maximum value vs. resolution of the DAC for several OSR.

certain conditions are met for the resolution of the DAC - different for each frequency. This means that the jitter power reduces only if $\hat{R}_{\Delta w}(0)/T_s^2$ resembles more and more the wanted continuous-time signal's derivative. In other words, OSR has an effect if sufficient resolution exists for a given frequency of the signal that new sample values can be created within two of the previous ones. So, it is possible that increasing the OSR will not give the expected jitter performance gain. Until saturation occurs, we observe a 3dB gain for every extra bit of resolution.

Finally, we are interested to identify the role of noise shaping in the jitter power. Recall that the PSD of the derivative of the discrete signal contains two parts. The first part is approximately equal to the PSD of the derivative of the original continuous-time signal $x(t)$. The second is the PSD of the derivative of the quantization noise. Since noise shaping moves the quantization power from the baseband to higher frequencies while keeping the signal unaffected, it increases effectively the second part . Hence, high order noise shaping is expected to have impact on the jitter noise generated by the DAC. In the best case, if discrete time filtering prior the DAC removes all the high frequency quantization power, the jitter power is dominated by the signal's derivative PSD. Such an approach was used in a practical implementation in [16].

2.2. Correlated jitter

In practice, most of the timing errors are correlated, however, due to complexity of modeling and mathematical methods they are simplified or non addressed at all. Here, applying the proposed method, we try to obtain insight in an emerging type of correlated jitter in DACs, the supply/substrate caused jitter [5]. It is the result of abrupt drain/injection of current from/to the supply/substrate during the activity of the digital blocks in the DAC. The problem is very difficult to model because of many factors that affect the jitter properties. In addition to the dependencies on the data, the technology (low/high ohmic substrate), the topology and the design of the DAC (e.g. separate power supplies, power supply rejection ratio of the clocked modules) random signals also appear. Our objective here is not to provide an accurate physical related model for the jitter, rather to exemplify that some intuition and quantitative results can be obtained with conceptual modeling.

We assume that the μ_m depend on the variations of the converted signal $\Delta w(m)$. Large variations means more noise because more units are switched on/off (decoder units, clocked buffers etc). The jitter amplitude is modulated by a white Gaussian variable [5] E with (m_E, σ_E) that exhibits white sense stationarity. A factor $K \approx 2^{N-1}$ is needed to normalize $\Delta w(m)$ over the full scale value of the converter. The delays become $\mu_m = \frac{\Delta w(m)}{K} E_m$ and for the sinusoidal input we approximate the time averaged probabilistic autocorrelation $r_\mu(q)$ of μ_m :

$$r_\mu(q) = \frac{\hat{R}_{\Delta w}(q)}{K^2}\sigma_E^2 = 2\sigma_E^2 \frac{A_1^2}{K^2}\sin^2(\frac{\omega_1 T_s}{2})\cos(\omega_1 q T_s) \quad (10)$$

The jitter effects split in two components, a white caused by the random E_m and a discrete added to the signal:

$$C_q(f,-f) = e^{\omega^2(r_\mu(q)-\sigma_E^2)} = |C(f)|^2(1+\sum_{\lambda=1}^{\infty}\frac{(\omega^{2\lambda} r_\mu(q)^\lambda}{\lambda!}) \quad (11)$$

with $|C(f)|^2 = e^{-\omega^2\sigma_E^2}$. After the calculations we end up in

$$C_q(f,-f) = |C(f)|^2 \sum_p I_p(M) e^{jp\omega_1 q T_s} \quad (12)$$

where $I_p(z)$ is the p-th order modified Bessel function of the 1st kind and $M = 8\pi^2 f^2 \sigma_E^2 \frac{A_1^2}{K^2}\sin^2(\frac{\omega_1 T_s}{2})$. Finally, using eq. (12) with (3), $\hat{R}(0)$ and $\hat{R}(m,q)$ we arrive in

$$S_s(f) = \frac{|C(f)|^2}{(\pi f T_s)^2}\sum_{p,q}\sum_{k=1,\text{odd}}^{\infty}\frac{A_k^2\sin^2(\pi f T_s)}{4}\cdot I_p(M)\delta(f \pm f_k - pf_1 - qf_s) \quad (13)$$

which describes the output spectrum.

The significance of this problem and the validity of the above is demonstrated with behavioral level simulations. First, in fig. 5 (a) and (b) we show the jitter and it's power spectrum based on the above modeling for a $N = 12$ bit DAC. The input signal is a full scale sinusoid with $f_1 = 0.05 f_s$ and the jitter values are kept very low to show the significance of the problem: E_m has (m_E, σ_E)=(1%, 1%) in percentage over T_s. This means that if *all* the elements are off (output value is zero) and *all* turn on to compose the full scale value, the resulting delay has variance σ_E^2. In fig. 5(c,d) the output spectrum before and after applying the jitter is shown. Fig. 5(d) shows clearly the additional harmonics and the white noise term added due to jitter. Notice the significance of the second order harmonic (-68dB) for $f_1 = 0.05 f_s$.

3. CONCLUSIONS

A general analysis on stochastic timing errors in DACs has been presented for (non)correlated errors. Typical examples have been demonstrated. While non-correlated errors induce only noise, correlated add discrete components in the power spectrum in addition to noise. Small values of either jitter types can substantially limit wide-band DAC performance. The presented general method embraces previously published work and extends towards the area of $\Sigma\Delta$ DACs and CT $\Sigma\Delta$ ADCs, revealing the nature of the tradeoff between oversampling ratio, DAC resolution and noise shaping.

[5] The random amplitude E can be related to the parameters of the voltage controlled oscillator (VCO) that creates this clock signal [5].

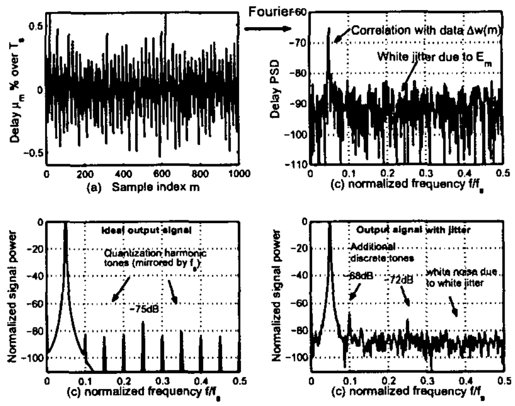

Fig. 5. Correlated jitter modeling and output signal spectrum.

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